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Test of the Universality of the Three-Body Efimov Parameter at Narrow Feshbach Resonances

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We measure the critical scattering length for the appearance of the first three-body bound state, or Efimov three-body parameter, at seven different Feshbach resonances in ultracold ³⁹K atoms. We study both intermediate and narrow resonances, where the three-body spectrum is expected to be determined by the nonuniversal coupling of two scattering channels. Instead, our observed ratio of the three-body parameter with the van der Waals radius is approximately the same universal ratio as for broader resonances. This unexpected observation suggests the presence of a new regime for three-body scattering at narrow resonances.

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The Efimov effect [1] was first described in the context of nuclear physics but is now explored also in atomic, molecular, and condensed-matter systems [2–5]. Recent experiments on ultracold atoms with Feshbach resonances [6–18] have opened up a new path to study the universal spectrum of three-body Efimov states. The resonant interaction is expected to give rise to a three-body potential scaling as $1/R^2$, where R is the hyperradius that parameterizes the moment of inertia of the system. This leads to an infinite series of trimer states with an universal geometrical scaling for the binding energies. For a finite, negative two-body scattering length a , the three-body potential has a long-range cutoff at $R \simeq |a|$, and only a finite number of bound states exist. The critical scattering length a_- for the appearance of the first Efimov state at the three-body threshold, often called the three-body parameter, was expected to be the only parameter to be influenced by nonuniversal physics, i.e., by the microscopic details of two- or even three-body forces [1,3]. While clear evidence of the universal scaling of the Efimov spectrum is still missing, recent experiments on identical bosons suggested that also a_- might be universal [19]. This surprising result has been interpreted in a recent series of theoretical studies [20–22]. The underlying idea is that the sharp drop in the two-body interaction potential at a distance of the order of the van der Waals radius R_{vdW} results in an effective barrier in the three-body potential at a comparable distance [22]. This prevents the three particles from coming sufficiently close to explore nonuniversal features of the interactions at short distances and leads to a three-body parameter set by R_{vdW} alone, $a_- \simeq -9.5R_{\text{vdW}}$ [19,21,22].

However, this scenario is realized only for the broad Feshbach resonances studied so far in most experiments, which can be described in terms of a single scattering channel, the so-called open channel. For narrow resonances,

one must instead take into account the coupling of the open and a second closed channel [23]. It has been shown that in this case a new length scale that depends on the details of the specific Feshbach resonance, the so-called intrinsic length R^* , must be introduced to parameterize the two-body scattering. The three-body potentials are also modified, with an expected deviation from the Efimovian dependence into $1/(R^*R)$ for distances $R < R^*$ [24]. This tends to reduce the depth of the three-body potential and leads to the non-universal result $a_- = -12.90R^*$ [24,25], which is much larger than that obtained for broad resonances. This prediction is valid only close to resonance, where $|a| \gg R^*$. It is still unclear how a_- scales in the intermediate regime of $|a| \simeq R^*$ or generally for resonances of intermediate widths. Various general models have been proposed [26–31], but they either are not fully predictive or give contradicting results.

In this Letter, we address this problem by performing an experimental study of three-body collisions in ultracold bosonic ³⁹K atoms, where we determine the three-body parameter a_- at several Feshbach resonances of intermediate or narrow width. In particular, our measurements probe for the first time the regime of very small resonance strengths, $s_{\text{res}} = 0.956R_{\text{vdW}}/R^* \simeq 0.1$, where R^* might be expected to be the relevant length scale that determines a_- . Surprisingly, we find values of a_- that are around the same $-9.5R_{\text{vdW}}$ measured for broad resonances, suggesting the existence of a novel intermediate regime of three-body scattering.

The investigation of closed-channel-dominated Feshbach resonances is particularly favored in ³⁹K, which has several resonances with moderate magnetic width Δ and relatively small background scattering length $-a_{\text{bg}} \simeq (20\text{--}30)a_0$ [32]. These parameters, together with the difference of the magnetic moments of the closed and open channels, $\delta\mu$,

determine the intrinsic length $R^* = \hbar^2/(ma_{\text{bg}}\Delta\delta\mu)$ [23], which can be related also to the on-resonance effective range (see the Supplemental Material [33]). In particular, we investigated seven different resonances with s_{res} in the range 0.1–2.8 in the three magnetic sublevels of the hyperfine ground state $F = 1$ [32].

A detailed description of the experimental setup is given elsewhere [34]. The three-body parameter was determined by finding the maximum of the three-body loss coefficient K_3 in the region of negative a at each Feshbach resonance, as in previous experiments [6–18]. In the presence of three-body losses, both the atom number N and temperature T evolve according to $dN/dt = -K_3\langle n^2\rangle N$ and $dT/dt = (K_3/3)\langle n^2\rangle T$, where $\langle n^2\rangle = (1/N) \int n(\vec{x})^3 d^3x$ is the mean square density [35]. The temperature increase is due to the preferential removal of atoms in the high-density region around the trap center. The typical starting condition was a noncondensed sample with $3\text{--}80 \times 10^4$ atoms in a temperature range of 20–400 nK, depending on the spin channel and Feshbach resonance (see the Supplemental Material [33]). The atoms were held in a purely optical trap (or with an additional magnetic confinement, depending on the specific resonance) at sufficiently low density to have a negligible mean-field interaction energy. Care was taken to have a trap depth sufficiently large to avoid evaporation associated to the heating. The samples were initially prepared at small negative a in proximity of the Feshbach resonances; the measurements started 10 ms after a was ramped to the final value in about 2 ms.

Figure 1 shows a typical evolution of N and T , as measured by absorption imaging after a free expansion. They were simultaneously fitted with

$$N(t) = N_0 \left/ \left(1 + \frac{3\beta^2}{\sqrt{27}} \frac{N_0^2}{T_0^3} K_3 t \right) \right|^{1/3}, \quad (1)$$

$$T(t) = T_0 \left(1 + \frac{3\beta^2}{\sqrt{27}} \frac{N_0^2}{T_0^3} K_3 t \right)^{1/9}. \quad (2)$$

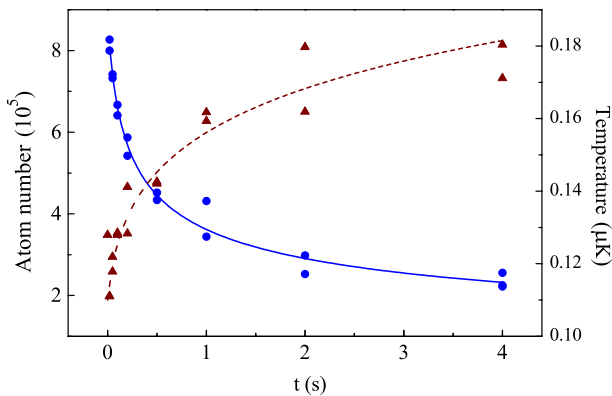


FIG. 1 (color online). Example of the time evolution of the atom number (circles) and temperature (triangles), fitted to Eq. (1) (solid line) and Eq. (2) (dashed line) to determine the three-body loss coefficient K_3 .

Here N_0 and T_0 are the initial atom number and temperature, respectively, and $\beta = (m\bar{\omega}^2/2\pi k_B)^{3/2}$, with $\bar{\omega}$ the mean trap frequency. In such a fit, one-body losses were neglected, since they occur on a much longer time scale.

Crucial ingredients for a reliable measurement of the K_3 dependence on the scattering length were an accurate calibration of the magnetic field B and the use of a high-quality coupled-channel (CC) model for $a(B)$, based on a large number of experimental observations for the positions and widths of the Feshbach resonances [32,33]. The centers and widths of the Feshbach resonances were redetermined in the present work, finding a good agreement with the theoretical ones. An additional confirmation of the CC model was derived from a direct measurement of the dimer binding energy at the two narrowest resonances by radio-frequency spectroscopy.

We observed for all Feshbach resonances a clear peak in K_3 in the region of $|a| = (600\text{--}1000)a_0$, as shown in Figs. 2 and 3. We compared the observations to the known

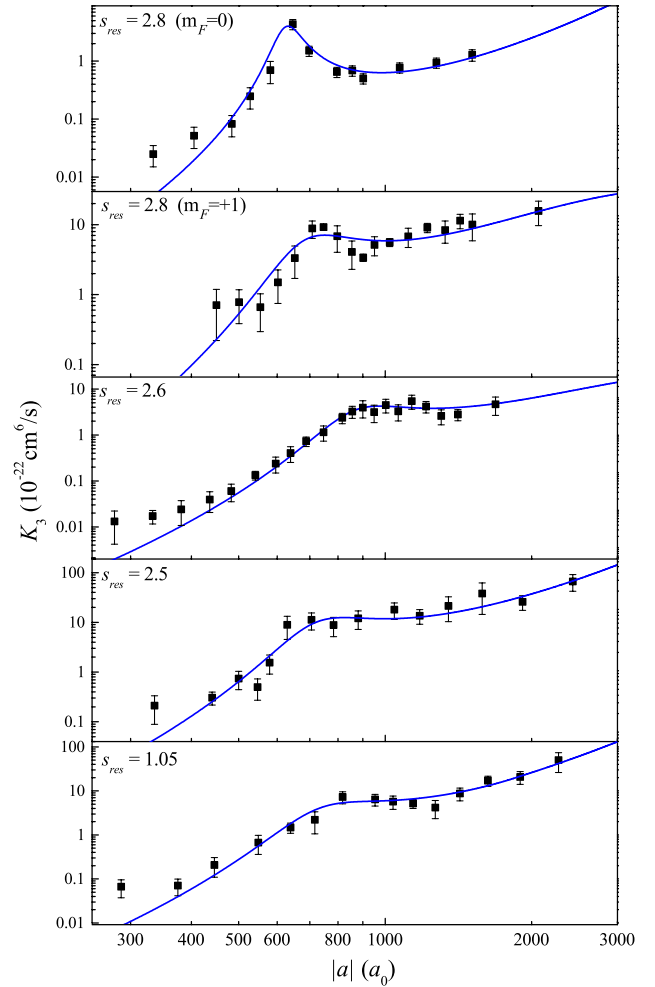


FIG. 2 (color online). Three-body loss rate measured in the proximity of five Feshbach resonances of intermediate strength (see Table I for the assignment of the spin state). The experimental data (squares) are fitted to Eq. (3) (solid line).

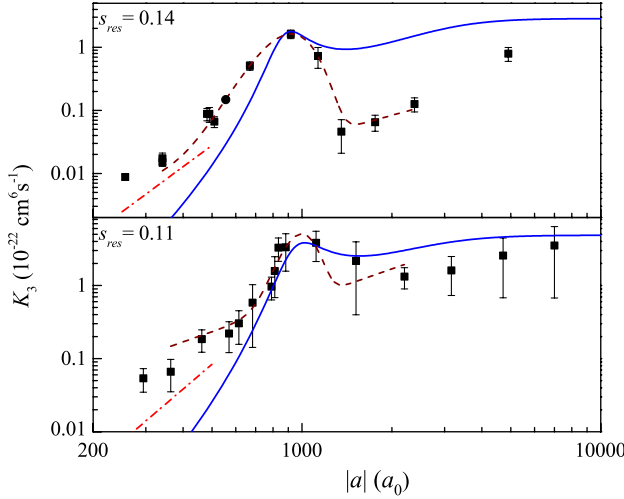


FIG. 3 (color online). Three-body loss rate measured in the proximity of two narrow Feshbach resonances in the $m_F = 0$ state. The experimental data (squares) are fitted with a Gaussian (dashed line) to determine a_- from the position of the loss maximum, and also compared with Eq. (3), using $\eta_- = 0.1$ (solid line). The dash-dotted lines provide a comparison to a $|a|^{7/2}$ behavior for low $|a|$.

relation for identical bosons at zero collision energy and in the zero-range approximation, for $a < 0$:

$$K_3(a) = 4590 \frac{3\hbar a^4}{m} \frac{\sinh(2\eta_-)}{\sin^2[s_0 \ln(a/a_-)] + \sinh^2 \eta_-}. \quad (3)$$

Here $s_0 \approx 1.00624$ is an universal constant, and η_- is the decay parameter which sets the width of the Efimov resonance and incorporates short-range inelastic transitions to deeply bound molecular states [3]. At the finite temperature of the experiment, there is a limitation in the maximum observable K_3 set by unitarity at $K_3^{\max} = 36\sqrt{3}\pi^2\hbar^5/(k_B T)^2 m^3$ [36,37]. Therefore, we fitted the data with an effective rate of the form $[1/K_3(a) + 1/K_3^{\max}]^{-1}$ [7,13,38], with a_- , η_- , and K_3^{\max} as fitting parameters. The experimental $K_3(a)$ for the five broadest resonances,

shown in Fig. 2, is in good agreement with Eq. (3), besides a multiplicative factor of the order of 3 that can be justified with the experimental uncertainty in the determination of the density (see the Supplemental Material [33]).

Also, the two narrowest Feshbach resonances feature a maximum in K_3 around $-1000a_0$, as shown in Fig. 3. There is, however, a slower background variation of K_3 with a , not reproduced by Eq. (3). It was shown that for narrow resonances one should expect a slower evolution in the regime $|a| < R^*$, with $K_3 \propto |a|^{7/2}$ [24], but also this does not seem to reproduce the data at small $|a|$. In the absence of a better model and in analogy with the broad resonances, we determined the position of the measured maximum in K_3 with a Gaussian fit, as shown in Fig. 3, and we interpreted it as the a_- parameter. As uncertainty, we conservatively took the $1/e^2$ half-width of the Gaussian.

For all the resonances in excited spin states, there is in principle also a contribution of two-body losses, which have a slower dependence on a [23]. While it was not possible to distinguish in a reliable way two- from three-body losses in the experiment, we have verified that the calculated two-body losses from the CC models are typically negligible, besides some large- a regions close to the two narrow resonances (see the Supplemental Material [33]).

A summary of our analysis is reported in Table I. For the calculation of $a(B)$, we used the experimentally determined Feshbach resonance centers B_0^{expt} and the resonance widths and the background scattering lengths from the CC model. The uncertainties in B_0^{expt} include those in the calibration of B and in the determination of B_0 from the loss resonances. Particular care was put in the determination of B_0 for the two narrowest resonances, where we found a rather good agreement between independent measurements of the atom losses and of the binding energy (see the Supplemental Material [33]). The uncertainties in a_- include the statistical uncertainties from the fit of the K_3 data and from the determination of $a(B)$. For the two narrowest resonances, the dominant source of uncertainty comes from the determination of B_0 . These two resonances are coupled, and $a(B)$ can be represented only over an

TABLE I. Theoretical and experimental parameters at Feshbach resonances in the m_F spin channels: measured resonance center B_0^{expt} ; intrinsic length R^* and strength s_{res} of the Feshbach resonances from the CC model; measured three-body parameter a_- and decay parameter η_- ; initial temperature T . For ^{39}K , $R_{\text{vdW}} = 64.49a_0$. Figures in parentheses represent one standard deviation.

m_F	B_0^{expt} (G)	R^* (a_0)	s_{res}	$-a_-$ (a_0)	η_-	T (nK)
0	471.0 (4)	22	2.8	640 (100)	0.065 (11)	50 (5)
+1	402.6 (2)	22	2.8	690 (40)	0.145 (12)	90 (6)
-1	33.64 (15)	23	2.6	830 (140)	0.204 (10)	120 (10)
-1	560.72 (20)	24	2.5	640 (90)	0.22 (2)	20 (7)
-1	162.35 (18)	59	1.1	730 (120)	0.26 (5)	40 (5)
0	65.67 (5)	456	0.14	950 (250)		330 (30)
0	58.92 (3)	556	0.11	950 (150)		400 (80)

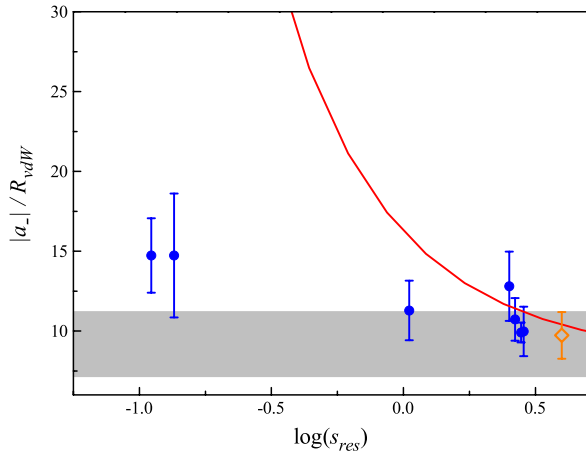


FIG. 4 (color online). Ratio of the measured $|a_-|$ to R_{vdW} as a function of the strength of the Feshbach resonances (filled circles). Our data are compared to predictions for $s_{\text{res}} \gg 1$ [21] (open diamond) and for generic s_{res} [26] (solid line). The gray shaded region shows the scatter in the experimental data for $|a_-|/R_{\text{vdW}}$ measured in other atomic species.

extended range of magnetic fields in terms of a two-pole expression containing two widths and a single a_{bg} [32]. The reported values of R^* are determined on resonance, from a comparison of the predictions of our CC calculation to a generalization of the quantum-defect model of Ref. [39] to the case of coupled resonances. The coupling causes a dependence of R^* on B , which is, however, limited to about 20% in the experimental range (see the Supplemental Material [33]).

We observe a whole range of values of η_- for the different Efimov resonances; this is probably a consequence of the different measurement temperatures but possibly also of the nonuniversal nature of η_- [3,40].

A comparison of the results in Table I leads to the striking conclusion that the three-body parameter a_- stays around values of the order of $-10R_{\text{vdW}}$ for all the Feshbach resonances explored in ^{39}K , including the ones with R^* as large as $\sim 600a_0$, hence much larger than R_{vdW} . We note that in the earlier measurement at the $m_F = 1$ resonance, we found two K_3 resonances at $|a| \simeq 700a_0$ and $|a| \simeq 1500a_0$, which we tentatively identified as a four- and a three-body resonance, respectively [7]. We now think that the previous resonance around $1500a_0$, which we no longer observe, was an artifact of the analysis of the limited time-dependent data, and we reassign the one around $700a_0$ as the three-body resonance (see the Supplemental Material [33]).

Figure 4 shows the measured $|a_-|/R_{\text{vdW}}$ as a function of s_{res} . We observe just a moderate deviation of our data from the mean value $9.73(3)$ measured for open-channel-dominated resonances [17,19,21,41] and also for other intermediate resonances [9–11,18,19,41]. This observation is far from the already mentioned prediction for narrow resonances [24,25], which indicates that the Efimov resonances should appear at scattering lengths that are

multiples of $a_- = -12.9R^*$ by a factor $\exp(\pi/s_0) \simeq 22.7$. One might note that this result is expected to be valid only in the limit of a scattering length larger than any other length scale, $|a| \gg R^* \gg |a_{\text{bg}}|$, where the three-body potential at large hyperradii $R > R^*$ has an Efimovian character [24]. The present experiment does not access this extreme limit but is in an intermediate regime also for the two narrowest resonances, which show indeed $R^* \simeq |a_-|$.

Other models for the three-body physics at Feshbach resonances of intermediate strength have been proposed [26–31]. The specific problem of connecting the results for the three-body parameter in the open-channel-dominated regime, where a_- is determined by R_{vdW} , and the closed-channel limit, where it is R^* which sets the scale for a_- , has been addressed recently [26,31], finding, however, considerably different results. In particular, the model of Ref. [26] predicts that a crossover between the two regimes of broad and narrow resonances would take place around $s_{\text{res}} \simeq 1$, as shown in Fig. 4. Additionally, the regime of $a_- = -12.9R^*$ should be reached only for excited Efimov states, while the first one has a slightly smaller $a_- = -10.3R^*$. Although an increase of $|a_-|$ with decreasing s_{res} might be present in the experimental data, there is a clear disagreement with such predictions. Experiments on ^7Li and ^{133}Cs have also measured similar values for a_- at three intermediate resonances with $s_{\text{res}} = 0.5$ –1 [10,11,18,19], indicating that this behavior might not be peculiar of ^{39}K . Also, a system without Feshbach resonances like metastable ^4He might be consistent with these results [42].

We note that for the two narrowest resonances $|a_-|$ is only a factor of 2 larger than R^* . This observation seems to indicate that the three-body potential can support a bound state that resides only in the region with hyperradius $R \leq 2R^*$. This is a regime that was not accessible in previous one-channel models, and a multichannel approach [43], possibly comprising the coupled-resonance aspect, will presumably be necessary to model the observations.

In conclusion, our study showed an apparent universal behavior of the three-body parameter on several different Feshbach resonances of the same atomic species, down to a resonance strength $s_{\text{res}} \simeq 0.1$. This gives important information on the three-body physics in this narrow-resonance regime, where an interplay of the open and the closed molecular channels is expected.

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